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The Unified Energy and Measurement Framework (UEMF) An engineering perspective for standardizing the modeling of dynamic systems

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ABSTRACT

Newtonian, Lagrange, and Hamiltonian dynamics are the cornerstones of physical mechanics, each offering a distinct perspective on describing motion. While Newton's system is based on Cartesian coordinates, Lagrange employs the concept of generalized coordinates in configuration space, and Hamiltonian mechanics uniquely describes the system using a set of first-order equations in phase space. In engineering, these concepts manifest in Kirchhoff's laws, state-variable representations, and Port-Hamilton models, which are pivotal tools in modern control and stability studies.

Despite their diverse tools, these descriptions are merely different languages describing the essence of a physical system: energy and its flow. This research develops the Unified Energy and Measurement Framework (UEMF) for modeling dynamical systems, a framework that unifies these perspectives and demonstrates their structural congruence. The framework aims to represent the system through a dual structure: the "energy equation," which describes the system in terms of its flow topology, energy stores, and dissipation mechanisms, and the "measurement equation," which acts as a gateway to translating internal energy variables into experimentally observable parameters. This framework thus combines the study of the system as an independent entity with its interaction with measurement instruments, paving the way for diverse advanced applications, including sensitivity analysis and quantum quantization.

Keywords: Unified Energy and Measurement Framework (UEMF), Symplectic Structure, Conjugate Energy Variables, Energy-Level System Modeling, Port Hamiltonian Dynamics, State Variable Equations, Hamiltonian Mechanics, Hessian Matrix, Dynamical System Topology, Quantum Quantization.

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إطار عمل الطاقة والقياس الموحد UEMF: نظرة هندسة لتوحيد نمذجة الأنظمة الديناميكية

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□ ملخّص □

تُعدّ ديناميكيات نيوتن ولاجرانج وهاملتون الركائز الأساسية للميكانيك الفيزيائي، حيث يقدم كل منها رؤية متمايزة لوصف الحركة، فبينما نيوتن يرتكز على الإحداثيات الديكارتية، يستثمر لاجرانج مفهوم الإحداثيات المعممة في فضاء التشكيل، في حين ينفرد ميكانيك هاملتون بوصف النظام عبر جملة معادلات من الدرجة الأولى في فضاء الطور. هندسياً تتجلى هذه المفاهيم في قوانين كيرشوف، وتمثيلات متغيرات الحالة، ونماذج بورت هاملتون، وهي أدوات محورية في التحكم الحديث ودراسة الاستقرار.

إن هذه التوصيفات، على تنوع أدواتها، ليست إلاّ لغات مختلفة تصف جوهر النظام الفيزيائي المتمثل في الطاقة وتدفقها. هذا البحث يستنبط إطار عمل الطاقة والقياس الموحد UEMF (Unified Energy and Measurement Framework)، لنمذجة الأنظمة الديناميكية، وهو إطار يوحّد هذه الرؤى مبيناً تطابقها البنيوي. يسعى الإطار لتمثيل النظام عبر بنية ثنائية: "معادلة الطاقة" المنوطة بوصف النظام من حيث طوبولوجيا التدفق ومخازن الطاقة وآليات التبديد، و"معادلة القياس" التي تعمل كبوابة لترجمة متغيرات الطاقة الداخلية إلى بارامترات قابلة للرصد المخبري. بذلك يجمع هذا الإطار بين دراسة النظام ككينونة مستقلة، ودراسته من حيث تفاعله مع أدوات القياس، مما يمهد الطريق لتطبيقات متقدمة متنوعة منها في تحليل الحساسية والتكميم الكمومي.

الكلمات المفتاحية: إطار عمل الطاقة والقياس الموحد UEMF، متغيرات الطاقة المترافقة، نمذجة الأنظمة على مستوى الطاقة، ديناميك بورت هاملتون، معادلات متغيرات الحالة، ميكانيك هاملتون، مصفوفة هيسيان، طوبولوجيا الأنظمة الديناميكية، التكميم الكمومي<

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1-Introduction

Newton presented physics with his three laws, through which we have been able to understand the physical world and mathematically describe motion. However, the problems solved by Newton's laws are confined to Cartesian coordinates. Lagrange followed, introducing a different philosophy based on the concept of the path of least effort (variational principle) to describe motion using generalized coordinates (position and velocity) in a space called the configuration space.

The Euler-Lagrange equation enabled the description of motion within these coordinates. Hamilton then came with a different perspective based on the principle of conservation of structure and generalized coordinates, along with generalized conjugate momentum, in a new space called the phase space. Through Hamilton's equations, we have been able to describe motion and understand the time evolution of physical systems [1,2]. The UEMF framework was inspired by Hamilton's principle of conservation of structure to formulate the energy equation.

Each of these dynamics contributed to a deeper understanding of physical phenomena. Newtonian mechanics and the concept of gravity enabled humanity to study everything from the smallest physical objects to planets and the movement of galaxies. Lagrange mechanics contributed to the characterization of mechanical systems and played a role in formulating Einstein's theory of relativity, which unleashed new concepts and insights, such as the concept of the fabric of space-time. Hamiltonian mechanics made a significant contribution to the characterization of quantum systems, giving rise to the field of quantum mechanics.

On the other hand, in the field of engineering, researchers developed various formulations to describe the behavior and temporal evolution of electrical systems. Kirchhoff's laws formed the cornerstone of analyzing and understanding the behavior of electrical systems. This was followed by the formulation of the state-variables method in state space [3], which contributed to the development of modern control systems, optimal control, and the analysis of electrical and mechanical circuits and various energy systems. More recently, researchers developed a coherent dynamic called Port Hamiltonian dynamics, which contributed to the advancement of control, modeling, and the study of stability in various physical energy systems [4].

Research Problem

The problem with this research lies in the diversity of these dynamics, their tools, and their purposes, and consequently, the diversity of their specializations. A physicist studies some of these dynamics, while an engineer studies a different aspect. Reconciling these dynamics is impossible for both physicists and engineers due to the differing specializations and areas of focus in some dynamics compared to others. Furthermore, these dynamical models do not differentiate between the internal energy state of a system and what we can measure.

Research Significance and Objectives

It was natural to search for a common root for these dynamics, considering the unified subject they study: energy and its flow—the language of nature. This is what this research will attempte by proposing the UEMF framework. This research starts with the equations of state variables in state space to model dynamical systems, and performs a coordinate transformation from the capacitor potential and inductor current in state space to the capacitor charge and inductor magnetic flux as conjugate energy

variables in phase space. As is known, a capacitor stores energy in an electric field, and an inductor stores energy in a magnetic field. Thus, energy is revealed in its original form.

This formulation is called the new equations of state variables. After that the new equations of state variables is founded corresponded to Hamilton's equations and Port-Hamilton's dynamics equations, the new equations of state variables represent the root from which the dynamics of the classical equations of state variables in state space, Hamilton's equations, and Port-Hamilton's equations in phase space, branch out. However, the formulation of the new equations of state still lacked the transformation of energy variables into parameters measurable in engineering laboratories. After adding the convertibility of energy variables (capacitor charge, inductor flux) to measurable parameters such as voltage and current, a complete framework emerged that combines the study of energy through its variables with the measurability of these variables by converting them into measurable parameters. This framework is called the **Unified Energy and Measurement Framework** (**UEMF**). This framework is not merely a coordinate transformation; rather, in some aspects, it represents a physical necessity for the transition from Newtonian and Kirchhoff mechanics to Schrödinger's. This formulation ensures a connection to observable engineering reality while providing a solid foundation for canonical quantum mechanics.

UEMF Definition

The UEMF framework consists of two equations: the **energy equation** and the **measurement equation**. The energy equation deals with the dynamics of a system in phase space using energy variables (capacitor charge and inductor flux in electrical systems, and displacement and momentum in mechanical systems). Through this equation, the topological structure is studied through which energy flows, its storage elements, and its dissipation mechanisms. The measurement equation, on the other hand, transforms energy variables into measurable parameters for practical laboratory use (such as voltage, current, force, and velocity).

The Research Organization

This introductory article will focus on defining the Unified Energy and Measurement Framework (UEMF) and demonstrating its alignment with the dynamics of physical systems. To this end, we will select three dynamical models: the equations of state variables, the Hamiltonian equations, and the Port-Hamilton representation. the UEMF framework will be adopted as a basis for modeling dynamical systems from a engineering perspective, and this article will outline a series of future research projects that will apply this framework. The research is organized as follows: In section 2, the research discuss the dynamics that describe systems according to a first-order differential equation, namely the equations of state variables, Hamiltonian equations, and Port-Hamilton equations. In section 3, this research will discuss an illustrative example demonstrating the correspondence and equivalence of these dynamics for the analysis of electrical and mechanical circuits.

In this section, a new formulation based on conjugate energy variables is formulated to describe the dynamics of systems. All other dynamics are derived from this new formulation.

In section 4, this new formulation into a comprehensive framework is dedicated for modeling dynamical systems. This framework is called the Unified Energy and Measurement Framework (UEMF). This research will explain in detail the of the

energy equation and measurement equations and the matrices included within the UEMF framework. This research will also discuss the Hamiltonian, the Hamiltonian gradient, and the Hessian matrix of the Hamiltonian and their relationship to this formulation in energy-conserving systems. In section 5, additional examples of dynamical modeling are provided for more complex electrical circuits (third-order systems). In section 6, this research will present future applications and areas of focus for the UEMF framework, and the research will conclude the research with a summary.

2- System Dynamics Representation

In this section, the dynamics representation of systems based on first-order differential equations are introduced within a space called a state space: state variable representation, Hamiltonian representation, and Port-Hamilton representation.

2-1- State Variable Representation

State variable representation is used to describe, analyze, and design dynamic systems, whether linear or nonlinear, time-invariant or time-varying, single-variable or multi-variable, single-input and single-output or multi-input and multi-output systems. It is used to model deterministic and random systems, as well as continuous and discrete systems. It is used to model the internal dynamic state and the external dynamic behavior of the system (the system's temporal evolution). Thus, state variables provide comprehensive information about the system's internal structure.

This type of representation allows the use of state variables as physical quantities, such as the voltage applied to a capacitor or the current flowing through an inductor. State variables can also be abstract mathematical variables, and they can be measurable or immeasurable [3,8]. State variable representation is used in many applications, primarily in the analysis and design of modern automated control systems, optimal control, and stability analysis [3, 8], and in the analysis of electrical circuits [8].

The state of a dynamic system is defined as the smallest set of time-dependent variables, called state variables. By setting up the initial internal states of the system at time $t = t_0$ according to the initial conditions $x_1(t_0)$, $x_2(t_0)$, ..., $x_n(t_0)$, and knowing these initial conditions along with the system's input values, the current and future temporal behavior of the system at any other time $t_0 \geq t$ can be fully determined within a space called the state space. The state space is an n-dimensional space with coordinate axes x_1, x_2, ..., x_n, and any state can be represented by a point in the state space.

The state variable algorithm is applied to transform an *n*-degree differential equation into an *n*-degree independent first-degree differential equation. The equations for the state variables of linear systems that do not change with time are written as follows [11]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \qquad x(t_0) = x_0 \end{aligned} \tag{1}$$

Where $x(t)$ is the state variables (nx1), $x'(t)$ is the time derivative of the state variable (nx1), $u(t)$ is the system input, and $y(t)$ is the system output. The first equation is called the state equation, while the second is called the output equation.

Matrix A is the state matrix, system matrix, or dynamic matrix (nxm). Matrix B is the input matrix (nxr), matrix C is the output matrix (mxn), and matrix D is the transition matrix, representing the direct connection between input and output (nxn).

Matrix *A* plays a crucial role in the analysis, design of automated control and optimal control systems. It allows for the analysis of controllability, observability, and system stability. The eigenvalues of matrix *A* are the system poles, and the matrix's rank determines the system's degree. It can be formulated in various ways, each with its own specific applications [4,8]. If the *LC* harmonic oscillator circuit is taken as shown in Figure (1) to illustrate writing the equations for the state variables, the voltage applied across the capacitor $v_C(t)$, and the electric current passing through the inductor $i_L(t)$ are chosen as state variables, .

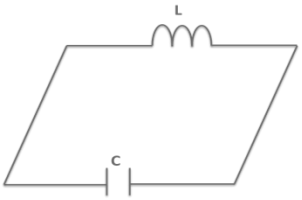


Figure (1): *LC* harmonic oscillator circuit.

The equations for state variables are written as follows:

$$\dot{x}_1(t) = \frac{dv_C(t)}{dt} = -\frac{1}{C} i_L(t), \qquad \dot{x}_2(t) = \frac{di_L(t)}{dt} = \frac{1}{L} v_C(t) \tag{2}$$

In matrix form, it is written as follows:

$$\begin{bmatrix} \frac{dv_C(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} \tag{3}$$

$$y(t) = v_C(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}, \quad or \quad y(t) = i_L(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}$$

These are first-order differential equations, while by applying Kirchhoff's law the oscillator circuit is represented by a second-order differential equation. This can be verified by applying Kirchhoff's voltage law (the sum of the applied voltages within a loop equals 0), resulting in the following equations:

$$L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = 0, \quad \Rightarrow \quad L\frac{d^2 i(t)}{dt^2} + \frac{i(t)}{C} = 0 \tag{4}$$

2-2- Hamilton's Equations

The story of representing the dynamics of systems began with Isaac Newton and his three well-known laws of motion, especially Newton's second law ($F = ma$), where *F* represents the force applied to the physical body, *m* represents the mass of the body, and *a* represents the acceleration of the body. Using Newtonian mechanics, scientists and researchers were able to formulate the equation of motion for mechanical systems. However, Newtonian mechanics relies primarily on Cartesian coordinates, also known as *x*-coordinates.

Lagrange mechanics then introduced a different dynamics to describe the motion of systems, based on the principle of least action and generalized coordinates (q_i) instead of Newtonian Cartesian coordinates. Lagrange presented his equation to describe motion which is suitable for describing motion in all coordinates and is therefore more general than Newton's equations. It can also describe motion in the presence of constraints of motion.

In Lagrange mechanics, any parameters or quantities (such as voltage and current in electrical circuits) can be used as generalized coordinates. These generalized coordinates are denoted as (q_1, q_2, …, q_n, q'_1, q'_2, …, q'_n), where qi represents the generalized coordinate and q'_i represents the generalized velocities corresponding to the generalized coordinate q_i. A key feature of Lagrange mechanics is the concept of degrees of freedom, which is the smallest set of independent generalized coordinates needed to fully describe the structure or state of a system. For a single particle moving in a vacuum, a degree of freedom is equal to 3, while for N particles moving in a vacuum, the degree of freedom becomes is equal to 3N (q_i(i = 1, 2, …, 3N)) [10, 11].

The space consisting of n generalized coordinates in a Lagrange representation is called the configuration space (C), and its dimension is *n* (*n*-dimensional space). If a vector *q* is plotted from the origin of the coordinates to a specific point in the configuration space (C), the time evolution of a given system is represented by a curve (the path of motion) in the configuration space, such that any point on this curve represents the state of the system at a specific instant.

The motion is described by a scalar Lagrange function (also called the Lagrangian), and is given by the following relation [1]:

$$L(q_1,\cdots,q_n,\dot{q}_1,\cdots,\dot{q}_n,t)=T-U \tag{5}$$

Where *T* represents kinetic energy and *U* represents potential energy.

The equation of motion, based on the Lagrange function is given by the following relation:

$$\frac{\partial L}{\partial q_i}-\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)=0 \qquad \forall i\in\{1,\cdots,\dim(C)\} \tag{6}$$

This equation is called the Euler-Lagrange equation [1], a second-order partial differential equation. Therefore, the Euler-Lagrange equation will be *n* second-order differential equations corresponding to the number of independent generalized coordinates.

Half a century after Lagrange, Hamilton introduced a new philosophy in the dynamics of systems and a new description of the equation of motion. Instead of the Euler-Lagrange equation describing motion with *n* second-order differential equations, Hamilton presented a description of motion consisting of *2n* first-order differential equations, utilizing the principle of conservation of structure. Hamilton also adopted new coordinates. Instead of the generalized coordinate q_i and the generalized velocity q'_i, Hamilton added a generalized momentum coordinate, p_i, to the generalized position coordinate q_i. Thus, the Hamiltonian function becomes:

$$H(q_1,\cdots,q_n,p_1,\cdots,p_n,t) \tag{7}$$

The generalized momentum is derived from the Lagrange function, and is given as follows:

$$p_i=\frac{\partial L}{\partial \dot{q}_i} \tag{8}$$

The Hamilton's function H (called the Hamiltonian) is defined as follows [2]:

$$H=\sum_{i=1}^{n} p_i\dot{q}_i-L \tag{9}$$

The Hamiltonian function is equal to the sum of the kinetic and potential energy, and therefore represents the total energy in the system. From the Hamiltonian function, a conservative standard system, which is written as follows:

$$H(q_1,\cdots,q_n,p_1,\cdots,p_n,t)=T+U \tag{10}$$

Based on the Hamiltonian function, and taking the derivative with respect to the coordinates q_i and p_i, the following two equations are obtain [13]:

$$\dot{q}=\frac{\partial H(q,p)}{\partial p},\qquad \dot{p}=-\frac{\partial H(q,p)}{\partial q} \tag{11}$$

These equations are called Hamilton's equations, and they can describe the motion of a system. It's important to note that the Euler-Lagrange equation and Hamiltonian equations lead to the same result for describing the motion of a system, but each dynamic has its own method.

The Lagrange dynamics relies on a configuration space to represent the generalized coordinates (q_i, q'_i), while Hamiltonian dynamics relies on a new space called the phase space. The dimension of this space is twice the dimension of the shaping space, i.e., a *2n*-dimensional phase space, extended by *2n* variables (q_1, q_2, …, q_n, p_1, p_2, ..., p_n). Each degree of freedom corresponds to a generalized coordinate with its conjugate generalized momentum; therefore, every point in the phase space is represented by the generalized coordinate q with its conjugate generalized momentum.

It is important to note the following difference: the Lagrange function (*L*) is a mathematical function with no physical meaning, while the Hamiltonian function (*H*) physically represents the total energy of the system. The Hamiltonian function is closely related to the concepts of symmetry and conservation of energy.

To illustrate the mechanism for describing the motion (dynamics of the system) using the Euler-Lagrange equation and Hamilton's equations, we will consider the circuit shown in Figure 1, which is an *LC* harmonic oscillator. To write the equation describing the operation of the circuit using Kirchhoff's laws, the Euler-Lagrange equation, and Hamilton's equations.

Applying Kirchhoff's voltage law (the sum of the applied potentials within a loop equals 0), and knowing that the current flowing in the circuit is equal to the change in charge *i(t) = dQ/dt*, the following equations are derived:

$$L\frac{di(t)}{dt}+\frac{1}{C}\int i(t)dt=0,\quad\Rightarrow\quad L\ddot{Q}+\frac{Q}{C}=0 \tag{11}$$

The resulting equation, representing the circuit's dynamics, is a second-order differential equation in terms of the capacitor's charge.

Kinetic energy is represented by the energy stored in the inductor as a magnetic field, and potential energy is represented by the energy stored in the capacitor as an electric field. The resulting Lagrange function is given by:

$$T=\frac{1}{2}L\dot{Q}^2=\frac{1}{2}\frac{\Phi^2}{L},\qquad V=\frac{1}{2}\frac{Q^2}{C}\quad\Rightarrow\quad L=T-V=\frac{1}{2}L\dot{Q}^2-\frac{1}{2}\frac{Q^2}{C} \tag{12}$$

Where *Q* is the charge of the capacitor, and Φ represents the magnetic flux in the inductor. The generalized coordinates of the Lagrange function are represented by the charge *Q* and the derivative of the charge (the current), i.e., L(*Q*, *Q'*, *t*).
Applying the Euler-Lagrange equation yields the following equation of motion:

$$\frac{\partial L}{\partial q_i}-\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)=0,\quad\Rightarrow\quad L\ddot{Q}+\frac{Q}{C}=0 \tag{13}$$

This is identical to the circuit equation obtained by applying Kirchhoff's potential law.

To formulate the motion using Hamilton's equations, first the generalized coordinates (coordinate and generalized momentum) is determined for the harmonic oscillator circuit. The charge of the capacitor Q as a generalized coordinate and the magnetic flux of the inductor Φ as its conjugate generalized momentum are chosen. To obtain the Hamiltonian function, the generalized momentum must be calculated as follows:

$$L = T - V = \frac{1}{2}L\dot{Q}^2 - \frac{1}{2}\frac{Q^2}{C} \tag{14}$$

$$p = \frac{\partial L}{\partial \dot{q}} \Rightarrow p = \frac{\partial L}{\partial \dot{Q}} = L\dot{Q} = LI = L\frac{\Phi}{L} = \Phi \Rightarrow p = \Phi$$

The Hamiltonian function resultes as follows (Eq. (10)):

$$H = p\dot{q} - L = \Phi\left(\frac{\Phi}{L}\right) - \frac{1}{2}L\left(\frac{\Phi^2}{L}\right) + \frac{1}{2}\frac{Q^2}{C} \quad \Rightarrow \quad H = \frac{1}{2}\frac{\Phi^2}{L} + \frac{1}{2}\frac{Q^2}{C} = T + V \tag{15}$$

Therefore, Hamilton's equations, which describe the behavior of the LC harmonic oscillator circuit, are produced as follows:

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{\partial H}{\partial \Phi} = \frac{\partial Q}{\partial t} \quad \Rightarrow \qquad \dot{Q} = \frac{\Phi}{L} \tag{16}$$

$$\dot{p} = -\frac{\partial H}{\partial q} = \frac{\partial H}{\partial Q} = \frac{\partial \Phi}{\partial t} \Rightarrow \qquad \dot{\Phi} = -\frac{Q}{C}$$

It is noteworthy that the first equation is the same as the generalized momentum equation. It is also noteworthy that the first Hamiltonian equation relates to the rate of change of charge, thus matching Kirchhoff's law of current, and the second equation relates to the rate of change of magnetic flux, thus matching Kirchhoff's law of potential. The negative sign (-) is physically explained by Lenz's law. A change in magnetic flux induces an induced magnetic field in the opposite direction, and the oscillation between the positive and negative signs of the Hamilton's equations causes the vibration in the harmonic oscillator circuit. The Hamilton's equations can be expressed in matrix form using the symplectic form [11]. If a column vector with a dimension of $2n$ elements, $z_1, z_2, ..., z_{2n}$, is defined as follows:

$$z_i = q_i, \qquad z_{n+i} = p_i, \qquad i = 1,2,...,n \tag{17}$$

The first n elements of the vector z represent n elements of the q-coordinate, and the last n elements of the vector z represent the generalized momentum p. The column vector of the Hamiltonian derivative with respect to the vector z is obtained as follows:

$$\left(\frac{\partial H}{\partial z}\right)_i = \frac{\partial H}{\partial q_i}, \qquad \left(\frac{\partial H}{\partial z}\right)_{n+i} = \frac{\partial H}{\partial p_i}, \qquad i = 1,2,...,n \tag{18}$$

Let's define the array J with dimensions $(2n)$x$(2n)$ as follows:

$$J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \tag{19}$$

Where J is a skew-symmetric matrix, 0 represents a zero matrix of dimensions nxn (zero matrix), and I represents an identity matrix of dimensions nxn (identity matrix). Matrix J is called the fundamental symplectic matrix. Hamilton's equations can be expressed in symmetric form as follows:

$$\dot{z} = J\frac{\partial H}{\partial z} \tag{20}$$

The symplectic matrix J have the following properties [2]:

$1-J^2=-I,\quad where I\ is\ the\ (2n)\times(2n)\ identity\ matrix$

$2-J^TJ=JJ^T=I\Rightarrow J^{-1}=J^T=-J\quad (J\ is\ an\ orthogonal\ matrix)$

$3-\det J=1.$

The matrix J is used to test whether a matrix has a symplectic structure. A matrix Ω is said to be symplectic if the following condition is met:

$$J^T\Omega J=\Omega \tag{21}$$

For the LC oscillatory circuit, we found that the Hamiltonian is given by the following:

$$H=\frac{1}{2}\frac{\Phi^2}{L}+\frac{1}{2}\frac{Q^2}{C} \tag{22}$$

Therefore, Hamilton's equations are written in symplectic and matrix form as follows:

$$\begin{bmatrix}\dot{Q}\\ \dot{\Phi}\end{bmatrix}=J\begin{bmatrix}\frac{\partial H}{\partial Q}\\ \frac{\partial H}{\partial \Phi}\end{bmatrix}\Rightarrow\begin{bmatrix}\dot{Q}\\ \dot{\Phi}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}\frac{Q}{C}\\ \frac{\Phi}{L}\end{bmatrix} \tag{23}$$

2-3- Port-Hamilton Dynamics

Port-Hamilton dynamics (pH) is used to model and analyze complex dynamical systems involving multiple physical systems (electrical, mechanical, thermal, etc.). Its applications include automation, optimal control, and stability analysis. PH dynamics is based on the variable flow and exchange of energy between different physical systems, extending the conventional state space into an energy state space. It utilizes the Dirac structure and Bond graphs to model various physical systems [4, 12].

An electrical circuit, according to the Port-Hamilton representation, is described by a set of first-order differential equations in the state space. These equations characterize the motion and time evolution of the system's dynamics. They are given as follows:

$$\dot{x}=\left[J(x)-R(x)\right]\frac{\partial H}{\partial x}(x)+g(x)u,\qquad y=g^T(x)\frac{\partial H}{\partial x}(x) \tag{24}$$

Where $J(x)$ represents a skew-symmetric matrix, also called a structure matrix (Dirac structure), with dimensions nxn, satisfying the property $J(x) = -J^T(x)$. x_1, x_2, ..., x_n represent the generalized coordinates in the n-dimensional state space, and x'_1, x'_2, ..., $x'n$ represent the derivatives of the generalized coordinates, responsible for organizing the energy flow between the storage elements (capacitor and inductor). $u(x)$ represents the system input, y represents the system output, and the matrix $g(x)$ represents the input matrix.

Where $R(x) = R^T(x) \geq 0$ represents the energy dissipation matrix (resistor structure matrix), which is a symmetric matrix.

For an LC oscillator circuit, assuming a current source $I(t)$ is connected in parallel with the oscillator circuit, and the output voltage is the voltage applied across the capacitor, the Port-Hamilton equations are given as follows:

$$\begin{bmatrix}\dot{Q}\\ \dot{\Phi}\end{bmatrix}=\left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}-\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\right)\begin{bmatrix}\frac{Q}{C}\\ \frac{\Phi}{L}\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}I(t),\qquad y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\frac{Q}{C}\\ \frac{\Phi}{L}\end{bmatrix} \tag{25}$$

The close relationship between port-Hamilton dynamics and the symplectic formulation of Hamilton's equations is noteworthy. The skew-symmetric matrix *J* represents the relationship between the capacitor and the inductor and how energy is exchanged between them.

3-Illustrative Example of Equivalence Between State-Variable Equations, Hamilton's Equations, and Port-Hamilton Equations:

In this section, the research will discuss, through an illustrative example (Figure 2), the equivalence of representing the equations of motion using state-variable equations, Hamilton's equations, and port-Hamilton dynamics. A formulation from the state-variable equations is generated in which the state variables are expressed in terms of the capacitor charge and the magnetic flux of the inductor, rather than the current through the capacitor and the voltage applied to the inductor, while maintaining the general representation of the state-variable equations. This formulation demonstrates the equivalence between the state-variable equations, Hamilton's equations, and port-Hamilton equations. The circuit shown in Figure (2) is a simple *RLC* circuit consisting of a resistor $G = 1/R$, an inductor L, and a capacitor C.

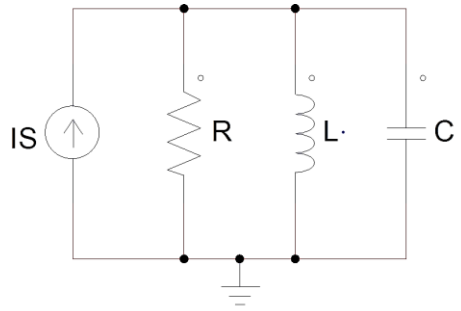


Figure (2): A simple RLC circuit.

The state variables, capacitor voltage and inductor current are chosen, the equations for the state variables are written as follows:

$$\dot{x}_1(t)=\frac{dv_C(t)}{dt}=-\frac{1}{C}i_L(t)-\frac{G}{C}v_C(t)+\frac{1}{C}I_S$$
$$\dot{x}_2(t)=\frac{di_L(t)}{dt}=\frac{1}{L}v_C(t) \tag{26}$$

It is written in matrix form as follows:

$$\begin{bmatrix}\frac{dv_C(t)}{dt}\\ \frac{di_L(t)}{dt}\end{bmatrix}=\begin{bmatrix}-\frac{G}{C} & -\frac{1}{C}\\ \frac{1}{L} & 0\end{bmatrix}\begin{bmatrix}v_C(t)\\ i_L(t)\end{bmatrix}+\begin{bmatrix}\frac{1}{C}\\ 0\end{bmatrix}I_S\ ,\qquad y(t)=v_C(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}v_C(t)\\ i_L(t)\end{bmatrix}+\begin{bmatrix}0\end{bmatrix}I_S \tag{27}$$

To write the equations for the state variables in terms of the capacitor charge and the magnetic flux of the inductor, the following transformation relationships result as:

$$v_C=\frac{Q}{C}\Rightarrow\quad\frac{dv_C(t)}{dt}=\frac{1}{C}\frac{dQ(t)}{dt}\Rightarrow\frac{dQ(t)}{dt}=C\frac{dv_C(t)}{dt}=i_C$$
$$i_L=\frac{\Phi}{L}\Rightarrow\quad\frac{di_L(t)}{dt}=\frac{1}{L}\frac{d\Phi(t)}{dt}\Rightarrow\frac{d\Phi(t)}{dt}=L\frac{di_L(t)}{dt}=v_L \tag{28}$$

By substituting the transformation relations into the equations for the state variables, the new state equations are obtained as follows:

$$\dot{x}_1(t)=\dot{Q}=\frac{dQ(t)}{dt}=\frac{\Phi}{L}-\frac{G}{C}Q+I_S$$
$$\dot{x}_2(t)=\dot{\Phi}=\frac{d\Phi(t)}{dt}=-\frac{Q}{C} \quad (29)$$

It is written in matrix form as follows:

$$\begin{bmatrix}\frac{dQ(t)}{dt}\\ \frac{d\Phi(t)}{dt}\end{bmatrix}=\underbrace{\begin{bmatrix}-\frac{G}{C} & -\frac{1}{L}\\ \frac{1}{C} & 0\end{bmatrix}}_{A_{new}}\begin{bmatrix}Q(t)\\ \Phi(t)\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}I_S \quad (30)$$

$$y_1(t)=Q(t)=\begin{bmatrix}\frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}Q(t)\\ \Phi(t)\end{bmatrix}+[0]I_S \text{ or } y_2(t)=\Phi(t)=\begin{bmatrix}0 & \frac{1}{L}\end{bmatrix}\begin{bmatrix}Q(t)\\ \Phi(t)\end{bmatrix}+[0]I_S$$

Note that the new state matrix has become A_{new}. The output $y_1(t)$ is chosen, which is the voltage applied to the capacitor, and $y_2(t)$ for the output voltage, which is the current through the inductor. Also note that the inputs of matrix B' now contain the value 1, and no longer contain the storage elements (C, L).

The formula for the equations of the new state variables is as follows:

$$\dot{z}(t)=A_{new}z(t)+B'u$$
$$y(t)=C'z(t)+D'u \quad (31)$$

This formulation represents the common background for these dynamics, demonstrating the correspondence between the representation of the state variable equations, the Hamilton's equations, and the Port Hamiltonian equations. From this formulation, this research will derive the Unified Energy Framework (UEMF).

The Hamiltonian H for an RLC circuit is given as follows:

$$H=T+V=E_C+E_L-E_R+E_S \quad (32)$$

Where T is kinetic energy, V is potential energy, E_C is the energy stored in the capacitor, E_L is the energy stored in the inductor, E_R is the energy dissipated through the resistor, and E_S is the energy source. It should be noted that the Hamiltonian, in the presence of dissipating elements such as the resistor, is no longer an energy-conserving system ($dH/dt \neq 0$).

The Hamiltonian H of the circuit is produced as follows:

$$H=\frac{Q^2}{2C}+\frac{\Phi^2}{2C}-\frac{1}{2R}v^2+\Phi I_S \quad (33)$$

The Hamilton's equations are given as follows:

$$\dot{q}=\frac{dH}{dp}, \qquad \dot{p}=-\frac{dH}{dq} \quad (34)$$

The generalized coordinate and the generalized momentum result as follows:

$$q=Q \quad generalized\ coordinate, \qquad p=\Phi \quad generalized\ momentum \quad (35)$$

Hamilton's equations are produced as follows:

$$\dot{q}=\frac{dH}{d\Phi}=\frac{dQ(t)}{dt}=\dot{Q}=\frac{\Phi}{L}-\frac{G}{C}Q+I_S, \qquad \dot{p}=-\frac{dH}{dQ}=\frac{d\Phi(t)}{dt}=\dot{\Phi}=-\frac{Q}{C} \quad (36)$$

As is evident, it corresponds to the equations of the new state variables (Eq. (30)) in terms of the capacitor charge and the inductor flux. It is obtained in matrix form as follows:

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix}=\begin{bmatrix}\dot{Q}\\ \dot{\Phi}\end{bmatrix}=\begin{bmatrix}\frac{dH}{d\Phi}\\ -\frac{dH}{dQ}\end{bmatrix}=\begin{bmatrix}-\frac{G}{C} & \frac{1}{L}\\ -\frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}Q(t)\\ \Phi(t)\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}I_S(t) \qquad (37)$$

To obtain a port-Hamilton representation, the matrix A_{new} is divided into two matrices [J-R], and multiply the state variable vector by the diagonal matrix Q_e, where Q_e = diag[1/C 1/L], thus obtaining the following equations:

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix}=\begin{bmatrix}\dot{Q}\\ \dot{\Phi}\end{bmatrix}=\left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}-\begin{bmatrix}G & 0\\ 0 & 0\end{bmatrix}\right)\begin{bmatrix}\frac{Q(t)}{C}\\ \frac{\Phi(t)}{L}\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}I_S(t), \qquad y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\frac{Q(t)}{C}\\ \frac{\Phi(t)}{L}\end{bmatrix} \qquad (38)$$

This is consistent with the Port-Hamilton equations (Eq. 24). It should be noted that the new matrix A_{new} contains the topological structure of the circuit represented by matrix J, and includes the dissipation matrix R.

Similarly, the mechanical circuit is taken as shown in Figure 3.

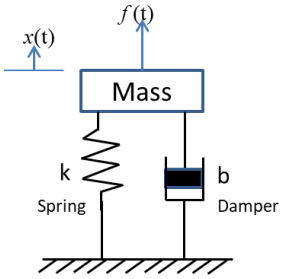


Figure (3): Mechanical circuit (equivalent to a serial RLC circuit).

The equations for the new state variables of the mechanical circuit are produced as follows:

$$\begin{bmatrix}x'\\ p'\end{bmatrix}=\begin{bmatrix}0 & \frac{1}{m}\\ -k & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}x\\ p\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}f(t) \qquad (39)$$

The Hamiltonian function for a mechanical circuit is written as follows:

$$H=\frac{k}{2}x^2+\frac{p^2}{2m}-\frac{b}{m}x-x\,f(t) \qquad (40)$$

Writing Hamilton's equations yields the following equations, which are identical to the equations Eq.(38).

$$x'=\frac{\partial H}{\partial p}=\frac{p}{m}, \qquad p'=-\frac{\partial H}{\partial x}=-kx-\frac{b}{m}+f(t) \qquad (41)$$

The Port-Hamilton equations are derived from Eq. (24) as follows:

$$z'=[J-R]\nabla H+g\cdot f_{ext} \quad\Rightarrow\quad \begin{bmatrix}x'\\ p'\end{bmatrix}=\left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & b\end{bmatrix}\right)\begin{bmatrix}kx\\ u\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}f(t) \qquad (42)$$

By giving this example from mechanical circuits to show the validity of formulating equations for new state variables on mechanical circuits in a manner similar to electrical circuits.

4-Unified Energy-Measurement Framework - UEMF

4-1- Derivation of Framework Equations

In this section, the Unified Energy and Measurement Framework (UEMF), is defined by explaining how to formulate it through the following steps:

1- The equations for the state variables in the state space of any linear system in classical logic are given as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (43)$$
$$y(t) = Cx(t) + Du(t)$$

Where x(t) is the state variable vector, and in electrical systems, it is $x(t) = [v_C, i_L]^T$. It is assume that $D = 0$ without loss of generality; there is no direct relationship between the input and output, which is the most common case in electrical circuits.

2- It is performed a coordinate transformation using the transformation equations (Eq. (28)), resulting in the following equations:

$$\dot{z}(t) = A_{new}z(t) + B'u(t) \qquad (44)$$
$$y(t) = C'z(t)$$

Where $z(t)$ is the vector of the new state variables, and the state variables in this case are the conjugate energy variables, which in electrical systems represent the charge of the capacitor and the magnetic flux of the inductor: $z(t) = [Q, \Phi]^T$. This formulation is called the new state equations, noting that the matrix B' no longer contains the energy storage elements. The energy storage elements are now confined to the new matrix A_{new} in the state equation, and they have moved to the matrix C' in the output equation.

This change in coordinates takes us into Hamiltonian space (phase space). Here, the Hamiltonian is given as follows

$$H = \frac{1}{2} z^T Q_e z \qquad (45)$$

Where the matrix Q_e is the Hessian containing the physical elements Q_e = diag[1/C, 1/L].

3- From Port-Hamilton system theory, It is known that the dynamics of any physical system can be decomposed according to the following relationship:

$$\dot{z} = [J - R)]\nabla H(z) \qquad (46)$$

4-Since $\nabla H = Q_e z$, and since the formulation of the equations for the new state variables matches Hamilton's formulation, the matrix A_{new} is given as follows:

$$A_{new} = [J - R]Q_e \qquad (47)$$

5-Note that the Port-Hamilton representation results from the state variable equations by partitioning the A_{new} system matrix into matrices J, R, and Q_e. Therefore, the A_{new} system matrix is the product of an interaction between two structures: the first structure [J-R], which represents the topological structure of the system (the conductivity across matrix J and the energy dissipation paths across matrix R); and the second structure, matrix Q_e, which represents the physics of the system and includes the energy-storing values of the elements. Substituting these values into the new state variable equations, we obtain the following:

$$\dot{z}(t) = A_{new}z(t) + B'u(t) = [J - R]Q_e z(t) + B'u(t) \qquad (48)$$

Where $z(t) = [Q, \Phi]^T$ is the conjugate energy variables, including the capacitor charge and the inductor flux, and $u(t)$ represents the input signal.

This equation is called the **energy equation**. It deals with the topological structure of energy flow and exchange through the *J* matrix, and the study of energy dissipation through the *R* matrix, which includes the values of the energy storage elements. The A_{new} matrix represents the topological structure, physical properties, and energy dissipation mechanism by encompassing the three matrices. The $\boldsymbol{A_{new}}$ **is called mother matrix**, and the three matrices (J, R, Q_e) are called the **child matrices**.

6-The output equation, in the classical state variable equations, is $y = C\,x(t)$. It is known that the *C* matrix does not contain energy storage elements; rather, its role is to determine the output measurement location in which branch of the circuit (capacitor voltage and inductor current). When moving to the formulation of the new state variables, the output relation becomes $y = C'\,z(t)$. Here, the energy storage elements move from matrix *B'* to matrix *C'*. To make matrix *C'* free of storage elements, we multiply it by the Hessian matrix (the storage element matrix) Q_e = diag[1/*C*, 1/*L*], thus returning it to its initial state in the classical state equations, i.e., translating the energy variables (capacitor charge and inductor flux) into measurable parameters (capacitor voltage and inductor current). Consequently, the output equation can be formulated as follows:

$$y = C' z_{out}(t) = Q_e z(t) \tag{49}$$

$z_{out}(t)$ is the measurable state variables (capacitor voltage and inductor current), and z(t) is the energy state variables (capacitor charge and inductor flux). The output y becomes either the capacitor voltage or the inductor current. Note the output equation in this framework is called the **measurement equation** because it translates the conjugate energy variables into measurable parameters in the laboratory. The measurement equations in this case are a transformation relationship from energy coordinates to measurable coordinates, and the transformation matrix that translates the energy variables into measurable parameters is the Hessian matrix $C_{out} = Q_e$, which is equal to the gradient of the Hamiltonian.

7-Thus, the equations of the UEMF framework, which connects the energy world and the measurement world, are formulated as follows:

$$\dot{z}(t) = A_{new} z(t) + B' u(t) = \left[J - R\right] Q_e\, z(t) + B' u(t) \qquad Energy equation \tag{50}$$

$$y(t) = C' z_{out}(t) = C_{out} z(t) = Q_e\, z(t) \qquad Measurement\ equation$$

As shown by the equations, the UEMF framework space is an energy-oriented state space. Clearly, the measurement equation is part of the energy equation, thus making the observer an integral part of the system, and the laws governing the energy equation are the same as those governing the measurement equation.

4-2- UEMF Framework Philosophy

Formulating the UEMF framework equations in this way, which places the energy variables (capacitor charge and inductor flux) in the energy equation and the voltage and current in the measurement equation, allows for the isolation of the circuit's form from its components. This constructs a transparent physical output equation, $y = Q_e\, z$, linking the world of energy to the world of measurement. It allows for a direct transition into the world of quantum mechanics. The energy equation is dedicated to studying the internal energy state of the system, since the associated energy state variables carry the energy information and the quantum identity of the system. The measurement equation, on the other hand, is dedicated to dealing with the

external world, transforming the energy variables and the hidden quantum state within the system into measurable physical quantities.

The energy equation, within its structure, it tells where the measurement equation lies. The product ($Q_e z$) in the energy equation is identical to the measurement equation; thus, we are dealing with a single equation in which the measurement equation is contained. This feature allows to understand that the observer becomes part of the system's equation via the output matrix (the Hessian matrix), without distorting the sanctity of the internal energy structure. Thus, the laws of physics remain the same in nature and in the experimental laboratory.

In this sense, the UEMF framework is based on a philosophy of simultaneous separation and connection. If each equation is considered independently, a complete separation between the realm of hidden energies is founded, inhabited by Hamiltonians through conjugate energy variables (the language of nature), and the world of physical measurements through tangible physical values such as voltage, current, displacement, and momentum (the language of the laboratory). However, if the framework is taken as an integrated system with its two equations together, it connects and bridges the gap between the world of energies and the world of measurements, translating the secrets of energy into the world of measurement laboratories.

It became clear that from the equations of the UEMF framework, the equations of the classical state variables are derived via the transformation equations (Eq. (28)), which are identical to the Hamilton's equations (energy equation). By partitioning the A_{new} matrix into the [*J-R*] matrices with the Hamiltonian gradient, we obtain the Port-Hamilton equations. It is known that the classical state equations are identical to Kirchhoff's laws for describing electrical circuits, and it is also known that the Hamilton's equations are identical to the Euler-Lagrange equation. Table (1) provides a detailed explanation of the functions of the UEMF framework matrices.

4-3- Comparison between the UEMF Framework and Other Dynamics

From the definition of the UEMF framework's structure, it is notable that the most important observations that illustrate the difference between the UEMF framework, the Port-Hamilton framework, and other dynamics:

- The UEMF framework retains all matrices (the mother matrix and the child matrices), and each matrix contains magical properties, which is surprising, as shown in Table (1). In contrast, the Port-Hamilton representation sacrifices the mother matrix in favor of the child matrices. It should be noted that the Q_e matrix is not explicitly present in the Port-Hamilton representation but is hidden within the Hamiltonian gradient. In the UEMF framework, it is adopted the opposite approach; that is, the Q_e matrix will appear in the energy equation, and the Hamiltonian gradient will be hidden behind it. This reveals the energy storage elements in the system (the nature of the system), and the importance of the Q_e matrix being present in the equation will become apparent when analyzing sensitivity.
- The Hamiltonian gradient does not appear in the UEMF energy equation in its abstract symbolic form, but rather manifests in the structure of the matrix Q_e and the state vector z(t).

Table (1): UEMF Framework Matrices and Their Functions

The Matrix	The Function
A_{new}	The system matrix A_{new}, the mother matrix of the child matrices J, R, and Q_e, represents the dynamic core of the system. It encompasses the topological and physical properties and includes the elements that dissipate the system's energy. It transfers some of its functions to the child matrices but retains its own. It is responsible for the system's rhythm through its intrinsic values, which determine the frequencies at which the system operates. Its asymmetry gives rise to vibrational motion through the exchange of positive and negative signals of values outside the main diameter. When applied to quantum mechanics, we find that creation and annihilation effects arise from it. Through these effects, it is responsible for the creation and annihilation of space (the quantum vacuum).
J	The topological structure matrix, responsible for the system's structure independently of the values of its constituent elements, determines the mechanisms of energy flow between components. Its skew-symmetry and symplectic structure determine the stability of the system's structure. In energy-conservative systems, the mother matrix A_{new} equals $A_{new} = JQ_e$, meaning that the matrix J transitions us from the dynamic state represented by the mother matrix A_{new} to the **energy-rest** state represented by the Hessian matrix Qe. Conversely, for $-JA_{new} = Q_e$, the matrix J transitions us from dynamic state **to energy-rest** (deriving the energy structure from the equation of motion) When applied to quantum quantization, it plays a fundamental role in Heisenberg's uncertainty principle, acting as a container for quantum information.
R	The energy dissipation matrix, responsible for energy loss in a system through its positive semi-defined property ($R \geq 0$), plays a significant role in the decay and damping of the system's response. It transforms the system from a closed, energy-conserving system to an open system subject to energy dissipation. In electrical systems, it is represented by the resistance R, and in mechanical systems by the damper b. When applied to quantum mechanics, it represents **breakdown** operators.
Q_e	The energy storage matrix, a positive diagonal symmetric matrix, contains the values of energy storage elements, such as capacitors and inductors in electrical systems, and spring and masses in mechanical systems, on its main diagonal. This matrix determines whether the system is electrical or mechanical. The $C_{out} = Q_e$ matrix is responsible for translating the energy variables in the energy equation into measurable parameters (the gradient of the Hamiltonian function) in the measurement equation. The C_{out} matrix acts as an observation matrix, translating the energy variables into current and voltage for electrical systems and into velocity and force for mechanical systems. Together with the energy variables, it determines the Hamiltonian function in linear systems. It is equivalent to the Hessian matrix of the Hamiltonian, whose eigenvalues allow us to investigate the presence or absence of an equilibrium point (the topography of the Hamiltonian function), thus playing a role in determining whether a system is stable or unstable.
B'	The input matrix, freed from its storage elements (capacitor and inductor or spring constant and mass), is now confined to the A_{new} matrix, leaving only the topological role of the B' matrix, which determines where the sources are connected. This matrix will play a role in determining the interaction Hamiltonian function and the interaction operator when applying the UEMF framework equations in quantum quantization.
C'	The output matrix, also freed from its storage elements after the coordinate transformation, is now limited to specifying the point at which the measurement will be performed. The state variables in the output equation are now the measurable parameters, such as current, voltage, speed, and force. We will retain the C' matrix in applications that require it, such as studying observability in modern control systems; otherwise, we will write the measurement equation in the form of the Hamiltonian gradient function in terms of the Hessian matrix and the energy variables $y = Q_e z$.
C_{out}	It represents a physical mirror of the Hessian matrix, appearing in the measurement equation, to ensure consistency between energy and measurement.

- Its function is relegated to the measurement equation, as its purpose is to translate energy variables into measurable parameters. In contrast, the Port-Hamilton representation reveals the Hamiltonian gradient in both equations, at the expense of the Q_e matrix, which is hidden behind the Hamiltonian gradient in the Port-Hamilton representation.
- The UEMF framework relies on a complete separation between studying the system from the perspective of energy and its variables, using the energy equation, and studying it from the perspective of measurability, using the measurement equation. The latter has become an integral part of the system's structure, facilitating control and stability analysis. This precision in distinguishing between the hidden energy world and the apparent measurement world is not observed in the Port-Hamilton framework.
- While the original state equations are powerful for analyzing and describing dynamical systems, they often remain abstract mathematical representations that require direct connection to energy physics.
- The Hamiltonian representation is masterful at describing and studying energy variables in phase space, but it doesn't tell us how to translate those variables into measurable values in electrical engineering laboratories.
- The UEMF framework exhibits an exceptional advantage not found in other dynamics frameworks. This advantage lies in the mirroring property between the energy and measurement equations. Part of the dynamic driver in the energy equation (the product $Q_e\, z$) is the same as that in the measurement equation. Mathematically, this means that the observer is no longer an outsider to the system, but rather an integral part of its fabric via the Hessian matrix Q_e. This cohesion provides a geometric explanation for the phenomenon of observer-system interaction and ensures that the physical laws remain unaltered, whether deep within the energy system or on the laboratory measurement platform. Thus, the UEMF framework achieves structural consistency, since the force driving the system (the energy gradient) is the same force measured in the laboratory.

4-4-Derivation of the Hamiltonian and Hessian gradients and the Hamiltonian function in terms of UEMF framework matrices

To derive the gradient of the Hamiltonian function in terms of UEMF framework matrices for energy-conservative systems, the *LC* harmonic oscillator circuit is taken. The new state equations are written as follows:

$$\begin{bmatrix} \frac{dH}{d\Phi} \\ \frac{dH}{dQ} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} Q(t) \\ \Phi(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_S(t) \tag{51}$$

By multiplying both sides of the equation by the fundamental cohesion matrix *J* (the skew-symmetry matrix), getting the following:

$$\underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{dH}{d\Phi} \\ \frac{dH}{dQ} \end{bmatrix}}_{\nabla H_{(Q,\Phi)}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} Q(t) \\ \Phi(t) \end{bmatrix} \Rightarrow \nabla H_{(z)} = \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} z = Q_e\, z \tag{52}$$

The left side of the equation represents the Hamiltonian gradient, and if adding the state variables with a vector $z = [Q(t) \quad \Phi(t)]^T$, resulting the following:

$$\nabla H_{(z)} = -J\, A_{new}\, z = J^T A_{new}\, z = Q_e\, z \tag{53}$$

Taking the second derivative of the Hamiltonian, obtaining the following Hessian matrix:

$$\nabla^2 H(q,p) = \begin{bmatrix} \frac{\partial^2 H}{\partial q^2} & \frac{\partial^2 H}{\partial q \partial p} \\ \frac{\partial^2 H}{\partial p \partial q} & \frac{\partial^2 H}{\partial p^2} \end{bmatrix} = -J\, A_{new} = J^T A_{new} = -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} = Q \tag{54}$$

The Hessian matrix is directly related to the system matrix A_{new} and the skew-symmetric symplectic matrix J. Therefore, the Hamiltonian can be written as a quadratic function as follows:

$$H = \frac{1}{2} z^T (-J\, A_{new}) z = \frac{1}{2} z^T J^T A_{new} z = \frac{1}{2} z^T Q\, z \tag{55}$$

It's important to note that when writing the Hamiltonian in terms of the matrix ($-JA_{new}$), the Hamiltonian tells us the law of motion, considering the mother matrix A_{new} as the engine of time evolution. By multiplying it by $-J$, it is projected the system's dynamics onto the energy container, and the energy is generated from the system's rotation in phase space.

However, when writing the Hamiltonian function in terms of the Hessian matrix Q_e, the function tells us the physical identity, considering the matrix Q_e as representing the energy stiffness or elasticity of the storage elements (capacitors, inductor, springs). Here, we are speaking in terms of static energy and potential energy, and the energy is represented by the properties of the element from which the system is made.

7-Example: RLC Circuit

In this example, Figure (4) represents an attempt to write the equations of the UEMF framework, and from them it is deduced: the classical state variables, the Hamiltonian equations, and the Port Hamiltonian representation.

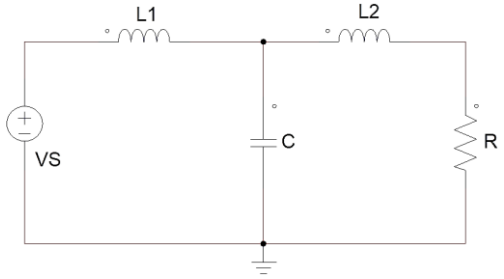


Figure (4): third-order electrical circuits

The state variable vector is the energy variable $z = [Q, \Phi_1, \Phi_2]^T$, the input is the voltage source $u = [V_1]$, and the output is the measurable values (capacitor voltage and inductor currents): $y = [v_C, i_{L1}, i_{L2}]^T$.

The UEMF framework equations are derived as follows:

$$\begin{bmatrix} \frac{dQ}{dt} \\ \frac{d\Phi_1}{dt} \\ \frac{d\Phi_2}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{L_1} & \frac{1}{L_2} \\ -\frac{1}{C} & 0 & -\frac{R}{L_2} \\ -\frac{1}{C} & 0 & 0 \end{bmatrix}}_{A_{new}} \begin{bmatrix} Q \\ \Phi_1 \\ \Phi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V_S \tag{56}$$

By substitution $A_{new} = [J - R]Q_e$, the power equation is produced as follows:

$$\begin{bmatrix} \frac{dQ}{dt} \\ \frac{d\Phi_1}{dt} \\ \frac{d\Phi_2}{dt} \end{bmatrix} = \underbrace{\left(\underbrace{\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}}_{J} - \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & R \\ 0 & 0 & 0 \end{bmatrix}}_{R} \right) \underbrace{\begin{bmatrix} \frac{1}{C} & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} \end{bmatrix}}_{Q_e}}_{A_{new}} \begin{bmatrix} Q \\ \Phi_1 \\ \Phi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V_S \tag{57}$$

The measurement equation is produced as follows:

$$y = \begin{bmatrix} v_C \\ i_{L_1} \\ i_{L_2} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{C} & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} \end{bmatrix}}_{C_{out}=Q_e} \begin{bmatrix} Q \\ \Phi_1 \\ \Phi_2 \end{bmatrix} \tag{57}$$

By adopting the transformation relationship (Eq. 28), the equations for the state variables in matrix form are produced as follows:

$$\begin{bmatrix} \frac{dv_C}{dt} \\ \frac{di_{L_1}}{dt} \\ \frac{di_{L_2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} & \frac{1}{C} \\ -\frac{1}{L_1} & 0 & 0 \\ -\frac{1}{L_2} & -\frac{R}{L_2} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_{L1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L_1} \\ 0 \end{bmatrix} V_S \tag{59}$$

To obtain Hamilton's equations, the generalized coordinates (Q_1, Q_2) are chosen where Q_1 represents the charge distributed within the first loop and Q_2 represents the charge distributed within the second loop. It is also chosen the generalized momentum (Φ_1, Φ_2), where Φ_1 represents the magnetic flux of the first inductor and Φ_2 represents the magnetic flux of the second inductor.

The Hamiltonian function (sum of kinetic and potential energy) is then derived as follows:

$$H = \frac{1}{2L_1}\Phi_1^2 + \frac{1}{2L_2}\Phi_2^2 + \frac{1}{2C}Q_1^2 + \frac{1}{2C}Q_2^2 - \frac{1}{C}Q_1Q_2 - \frac{1}{RL_2}\Phi_2 + Q_1V_S \tag{60}$$

Considering that $i_C = i_{L1} - i_{L2}$, and that the charge of the capacitor $Q_C = Q_2 - Q_1$, the Hamiltonian equations are derived, which are identical to Eq. 56.

The Port-Hamilton equations are derived directly by partitioning the matrix A_{new} into the matrices [J - R] as follows:

$$\begin{bmatrix} \frac{dQ}{dt} \\ \frac{d\Phi_1}{dt} \\ \frac{d\Phi_2}{dt} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}}_{J} - \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & R \\ 0 & 0 & 0 \end{bmatrix}}_{R} \right) \begin{bmatrix} \frac{Q}{C} \\ \frac{\Phi_1}{L_1} \\ \frac{\Phi_2}{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V_S \tag{61}$$

6- Prospects for UEMF Applications

Since this research demonstrated the correspondence between the UEMF framework and other dynamics, the application scope of these dynamics necessarily extends to the application scope of the UEMF framework.

It is possible to outline a path for future research on the UEMF framework in the following areas:

1- System topology engineering and structural stability through the properties of the J matrix. This is achieved through Graph theory. Given that the determinant of the A_{new} matrix is zero ($\det(A_{new}) = 0$), if the state variables are dependent, this indicates a flaw in the topological structure and the energy flow mechanism represented by the J matrix within the mother A_{new} matrix structure. Therefore, before quantum quantization, the integrity of the topological structure represented by the J matrix must be examined. This examination is crucial because the independence of the state variables is a prerequisite for correctly defining the degrees of freedom before proceeding to quantization. When quantum quantization is applied, the J matrix will play a fundamental role in Heisenberg's uncertainty principle.

2- Quantum quantization, because the UEMF framework models the system at the energy variable level, which in turn leads to a smooth transition to the quantum level, especially for circuits that rely on superconductivity in constructing the quantum bit [5,6]. This also facilitates the direct construction of the Hamiltonian from the circuit diagram. In our subsequent discussion, it is clear that the energy equation is the gateway to quantum mechanics, while the measurement equation represents the measurement gateway (or the measurement process). It describes how the wave function collapses, or how information is extracted from the quantum system into classical measuring instruments. It serves as the bridge that translates energy probabilities in the quantum world into definite measurement results in the classical world. This research will demonstrate that the A_{new} matrix is not merely a stability matrix, but rather a disguised classical Hamiltonian.

3. Re-examining controlability, obervability, stability, and order realization in modern control systems, in light of the adoption of conjugate energy variables and the A_{new} matrix (where eigenvalues remain constant, but eigenvectors change due to coordinate changes, without altering the system's dynamics), and in light of the changes in the structure of matrices B' and C'. It is clear that in some control systems, the B' matrix is sufficient to define controlability, and the $Q_e = C_{out}$ matrix is sufficient to define obervability.

4. Sensitivity analysis to study system behavior under the influence of changes in the values of its constituent elements, such as manufacturing defects and varying noise conditions. This research will find that Tellgen's theorem and the concept of the conjugate circuit are perfectly compatible with the UEMF framework because they rely on power conservation. Since the Q_e matrix, which represents the values of the elements, appears explicitly in both the energy equation and the measurement equation, calculating the derivatives (the sensitivity of the output to a change in an element, dy/dQ_e) becomes a straightforward algebraic process.

5. Modeling complex dynamic systems that incorporate different physical systems.

6. Analysis of Nonlinear Systems: In this research section, this research will demonstrate the relationship between the UEMF framework equations and the Brayton-Moser equations.

7. Applications of the framework in the field of optimal control, and the advantages it offers in the design of controllers and observer systems.

6- Conclusion

In this foundational research, the Unified Energy and Measurement Framework (UEMF) is presented as a mathematical and philosophical structure aimed at bridging the historical gap between different modeling languages in physics and engineering. It is demonstrated that the multiplicity of dynamical formulations, from Newton and Lagrange to Hamilton, Port-Hamilton, Kirchhoff, and state space, is merely a variation in the language used to describe a single, fundamental truth of nature: energy and its flow.

By decomposing the mother matrix A_{new} into its child matrices (J, R, Q), it is possible to isolate the topological structure from the physical properties, allowing us to visualize the dynamic dance of the system with unprecedented clarity. The most distinctive feature of the UEMF framework is its harmonious duality: the energy equation describes the system's being and essence in phase space, while the measurement equation acts as a cognitive gateway, translating these latent properties into experimentally observable variables, thus transforming the observed element from an external component into an integral and stable part of the equation's structure.

Our findings regarding the alignment of this framework with classical dynamics, and its high diagnostic capability for system stability across energy terrains, open up broad prospects for redefining controlability and obsevability concepts from a purely energy-based perspective.

In conclusion, the UEMF framework is not merely a mathematical transformation of coordinates, but a necessary precursor to the transition from classical geometry to quantum technologies. The next research will be the first step in this direction, where we will utilize this framework to study canonical quantum quantization and quantum entanglement of superconducting circuits, to see how our matrices in this research translate into quantum leaps that will govern the future of computing and measurement.

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